In this section, you will study the relationships between the volumes of gases that react with each other. You will also learn about the relationship between molar amount of gas and volume, and a single gas law that unifies all the basic gas laws into a single equation.

**Main Idea**

**Gases react in whole-number ratios.**

In the early 1800s, French chemist Joseph Gay-Lussac studied gas volume relationships involving a chemical reaction between hydrogen and oxygen. He observed that 2 L of hydrogen can react with 1 L of oxygen to form 2 L of water vapor at constant temperature and pressure.

\[
\text{hydrogen gas} \quad + \quad \text{oxygen gas} \quad \rightarrow \quad \text{water vapor} \\
2 \text{ L (2 volumes)} \quad 1 \text{ L (1 volume)} \quad 2 \text{ L (2 volumes)}
\]

In other words, this reaction shows a simple and definite 2 : 1 : 2 relationship between the volumes of the reactants and the product. Two volumes of hydrogen react with 1 volume of oxygen to produce 2 volumes of water vapor. The 2 : 1 : 2 relationship for this reaction applies to any proportions for volume—for example, 2 mL, 1 mL, and 2 mL; 600 L, 300 L, and 600 L; or 400 cm³, 200 cm³, and 400 cm³.

Gay-Lussac also noticed simple and definite proportions by volume in other reactions of gases, such as in the reaction between hydrogen gas and chlorine gas.

\[
\text{hydrogen gas} \quad + \quad \text{chlorine gas} \quad \rightarrow \quad \text{hydrogen chloride gas} \\
2 \text{ L (2 volumes)} \quad 1 \text{ L (1 volume)} \quad 2 \text{ L (2 volumes)}
\]

In 1808, in what is known today as Gay-Lussac’s law of combining volumes of gases, the scientist summarized the results of his experiments by stating that at constant temperature and pressure, the volumes of gaseous reactants and products can be expressed as ratios of small whole numbers. This simple observation, combined with the insight of Avogadro, provided more understanding of how gases react and combine with each other.
Equal volumes of gases under the same conditions contain equal numbers of molecules.

Recall an important point of Dalton’s atomic theory: atoms are indivisible. Dalton also thought that the particles of gaseous elements exist in the form of isolated single atoms. He believed that one atom of one element always combines with one atom of another element to form a single particle of the product. However, some of the volume relationships observed by Gay-Lussac could not be accounted for by Dalton’s theory. For example, in reactions such as the formation of water vapor, mentioned on the preceding page, it would seem that the oxygen atoms involved would have to divide into two parts.

In 1811, Avogadro found a way to explain Gay-Lussac’s simple ratios of combining volumes without violating Dalton’s idea of indivisible atoms. He did this by rejecting Dalton’s idea that reactant elements are always in monatomic form when they combine to form products. He reasoned that these molecules could contain more than one atom. Avogadro’s law states that equal volumes of gases at the same temperature and pressure contain equal numbers of molecules. Figure 3.1 illustrates Avogadro’s law. It follows that at the same temperature and pressure, the volume of any given gas varies directly with the number of molecules. Avogadro’s law also indicates that gas volume is directly proportional to the amount of gas, at a given temperature and pressure, as shown in the following equation:

Here, \( n \) is the amount of gas, in moles, and \( k \) is a constant.

**Avogadro’s law** \[ V = kn \]

Avogadro’s reasoning applies to the combining volumes for the reaction of hydrogen and oxygen to form water vapor. Dalton had guessed that the formula of water was HO, because this formula seemed to be the most likely formula for such a common compound. But Avogadro’s reasoning established that water must contain twice as many H atoms as O atoms, consistent with the formula \( \text{H}_2\text{O} \). As shown below, the coefficients in a chemical reaction involving gases indicate the relative numbers of molecules, the relative numbers of moles, and the relative volumes.

\[
2\text{H}_2(g) + \text{O}_2(g) \rightarrow 2\text{H}_2\text{O}(g)
\]

<table>
<thead>
<tr>
<th></th>
<th>(2) molecules</th>
<th>(1) molecule</th>
<th>(2) molecules</th>
</tr>
</thead>
<tbody>
<tr>
<td>mol</td>
<td>(2)</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>vol</td>
<td>(2)</td>
<td>(1)</td>
<td>(2)</td>
</tr>
</tbody>
</table>

The simplest hypothetical formula for oxygen indicated two oxygen atoms, which turns out to be correct. The simplest possible molecule of water indicated two hydrogen atoms and one oxygen atom per molecule, which is also correct. Experiments eventually showed that all elements that are gases near room temperature, except the noble gases, normally exist as diatomic molecules.

**CRITICAL THINKING**

**Conclude** By observing the figures above, what statement could you make about what these three gases at STP have in common?
Hydrogen gas
1 Volume
1 Molecule

Chlorine gas
1 Volume
1 Molecule

Hydrogen chloride gas
2 Volumes
2 Molecules

Consider the reaction of hydrogen and chlorine to produce hydrogen chloride, illustrated in Figure 3.2. According to Avogadro’s law, equal volumes of hydrogen and chlorine contain the same number of molecules. Avogadro’s idea of diatomic gases applies to this reaction also. He concluded that the hydrogen and chlorine components must each consist of two or more atoms joined together. The simplest assumption was that hydrogen and chlorine molecules are composed of two atoms each. That assumption leads to the following balanced equation for the reaction of hydrogen with chlorine.

\[
\text{H}_2(g) + \text{Cl}_2(g) \rightarrow 2\text{HCl}(g)
\]

The simplest hypothetical formula for hydrogen chloride, HCl, indicates that the molecule contains one hydrogen atom and one chlorine atom. Given the ratios of the combined volumes, the simplest formulas for hydrogen and chlorine must be H$_2$ and Cl$_2$, respectively.

**Main Idea**

All gases have a volume of 22.4 L under standard conditions.

Recall that one mole of a molecular substance contains a number of molecules equal to Avogadro’s constant (6.022 \( \times \) 10$^{23}$). One mole of oxygen, O$_2$, contains 6.022 \( \times \) 10$^{23}$ diatomic oxygen molecules and has a mass of 31.9988 g. One mole of helium, a monatomic gas, contains the same number of helium atoms and has a mass of 4.002 602 g.

According to Avogadro’s law, one mole of any gas will occupy the same volume as one mole of any other gas at the same temperature and pressure, despite mass differences. The volume occupied by one mole of a gas at STP is known as the standard molar volume of a gas. It has been found to be 22.414 10 L. For calculations in this book, we use 22.4 L as the standard molar volume.

Knowing the volume of a gas, you can use 1 mol/22.4 L as a conversion factor to find the number of moles, and therefore the mass, of a given volume of a given gas at STP. You can also use the molar volume of a gas to find the volume, at STP, of a known number of moles or a known mass of a gas.
Calculating with Avogadro’s Law

**Sample Problem G**

a. What volume does 0.0685 mol of gas occupy at STP?
b. What quantity of gas, in moles, is contained in 2.21 L at STP?

**SOLVE**

**a.** Multiply the amount in moles by the conversion factor, \( \frac{22.4 \text{ L}}{1 \text{ mol}} \).

\[
V = 0.0685 \text{ mol} \times \frac{22.4 \text{ L}}{1 \text{ mol}} = 1.53 \text{ L}
\]

**b.** Multiply the volume in liters by the conversion factor, \( \frac{1 \text{ mol}}{22.4 \text{ L}} \).

\[
\text{Moles} = 2.21 \times \frac{1 \text{ mol}}{22.4 \text{ L}} = 0.0987 \text{ mol}
\]

**Practice**

1. At STP, what is the volume of 7.08 mol of nitrogen gas?
2. A sample of hydrogen gas occupies 14.1 L at STP. How many moles of the gas are present?

**MAIN IDEA**

In a chemical equation, the coefficients can indicate moles, molecules, or volume.

You can apply the discoveries of Gay-Lussac and Avogadro to calculate the stoichiometry of reactions involving gases. For gaseous reactants or products, the coefficients in chemical equations not only indicate molar amounts and mole ratios but also reveal volume ratios, assuming conditions remain the same. For example, consider the reaction of carbon monoxide with oxygen to give carbon dioxide.

\[
2\text{CO}(g) + \text{O}_2(g) \rightarrow 2\text{CO}_2(g)
\]

2 molecules 1 molecule 2 molecules
2 mol 1 mol 2 mol
2 volumes 1 volume 2 volumes

The possible volume ratios can be expressed in the following ways.

<table>
<thead>
<tr>
<th></th>
<th>2 volumes CO</th>
<th>or</th>
<th>1 volume O₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1 volume O₂</td>
<td></td>
<td>2 volumes CO</td>
</tr>
<tr>
<td>b</td>
<td>2 volumes CO₂</td>
<td>or</td>
<td>2 volumes CO</td>
</tr>
<tr>
<td></td>
<td>2 volumes CO₂</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>1 volume O₂</td>
<td>or</td>
<td>2 volumes CO₂</td>
</tr>
<tr>
<td></td>
<td>2 volumes CO₂</td>
<td></td>
<td>1 volume O₂</td>
</tr>
</tbody>
</table>
Gas Stoichiometry

Sample Problem H  Propane, C₃H₈, is a gas that is sometimes used as a fuel for cooking and heating. The complete combustion of propane occurs according to the following balanced equation.

\[ \text{C}_3\text{H}_8(g) + 5\text{O}_2(g) \rightarrow 3\text{CO}_2(g) + 4\text{H}_2\text{O}(g) \]

(a) What will be the volume, in liters, of oxygen required for the complete combustion of 0.350 L of propane? (b) What will be the volume of carbon dioxide produced in the reaction? Assume that all volume measurements are made at the same temperature and pressure.

1. **Analyze**

   **Given:**
   
   balanced chemical equation
   
   \( V \) of propane = 0.350 L

   **Unknown:**
   
   a. \( V \) of O₂ in L
   
   b. \( V \) of CO₂ in L

2. **Plan**

   a. \( V \) of \( \text{C}_3\text{H}_8 \rightarrow V \) of O₂
   
   b. \( V \) of \( \text{C}_3\text{H}_8 \rightarrow V \) of CO₂

   All volumes are to be compared at the same temperature and pressure. Therefore, volume ratios can be used like mole ratios to find the unknowns.

3. **Solve**

   a. \( V \) of O₂ = \( 0.350 \text{ L} \text{C}_3\text{H}_8 \times \frac{5 \text{ L} \text{O}_2}{1 \text{ L} \text{C}_3\text{H}_8} = 1.75 \text{ L} \text{O}_2 \)
   
   b. \( V \) of CO₂ = \( 0.350 \text{ L} \text{C}_3\text{H}_8 \times \frac{3 \text{ L} \text{CO}_2}{1 \text{ L} \text{C}_3\text{H}_8} = 1.05 \text{ L} \text{CO}_2 \)

4. **Check Your Work**

   Each result is correctly given to three significant figures. The answers are reasonably close to estimated values of 2, calculated as 0.4 \( \times \) 5, and 1.2, calculated as 0.4 \( \times \) 3, respectively.

**Practice**  Answers in Appendix E

1. Assuming all volume measurements are made at the same temperature and pressure, what volume of hydrogen gas is needed to react completely with 4.55 L of oxygen gas to produce water vapor?

2. What volume of oxygen gas is needed to react completely with 0.626 L of carbon monoxide gas, CO, to form gaseous carbon dioxide? Assume all volume measurements are made at the same temperature and pressure.

3. Nitric acid can be produced by the reaction of gaseous nitrogen dioxide with water, according to the following balanced chemical equation.

\[ 3\text{NO}_2(g) + \text{H}_2\text{O}(l) \rightarrow 2\text{HNO}_3(l) + \text{NO}(g) \]

If 708 L of NO₂ gas react with water, what volume of NO gas will be produced? Assume the gases are measured under the same conditions before and after the reaction.
Pressure, volume, and temperature are related to the number of moles of a gas.

You have learned about equations describing the relationships between two or three of the four variables—pressure, volume, temperature, and moles—needed to describe a gas sample. All the gas laws you have learned thus far can be combined into a single equation. The ideal gas law is the mathematical relationship among pressure, volume, temperature, and the number of moles of a gas. It is the equation of state for an ideal gas, because the state of a gas can be defined by its pressure, volume, temperature, and number of moles. It is stated as shown below, where \( R \) is a constant.

\[
P V = n RT
\]

The ideal gas law reduces to Boyle’s law, Charles’s law, Gay-Lussac’s law, or Avogadro’s law when the appropriate variables are held constant.

The number of molecules or moles present will always affect at least one of the other three quantities. The collision rate of molecules per unit area of container wall depends on the number of molecules present. If the number of molecules is increased for a sample at constant volume and temperature, the collision rate increases. Therefore, the pressure increases, as shown by the model in Figure 3.3a. Consider what would happen if the pressure and temperature were kept constant while the number of molecules increased. According to Avogadro’s law, the volume would increase. As Figure 3.3b shows, an increase in volume keeps the pressure constant at constant temperature. Increasing the volume keeps the collision rate per unit of wall area constant.

**FIGURE 3.3**

The Ideal Gas Law

---

**CRITICAL THINKING**

**Analyze** When volume and temperature are constant, and the number of molecules increases, what happens to the gas pressure?

**Analyze** When pressure and temperature are constant, and the number of molecules increases, what happens to the gas volume?
The ideal gas law relates pressure to volume to temperature.

In the equation representing the ideal gas law, the constant \( R \) is known as the ideal gas constant. Its value depends on the units chosen for pressure, volume, and temperature. Measured values of \( P \), \( V \), \( T \), and \( n \) for a gas at near-ideal conditions can be used to calculate \( R \). Recall that the volume of one mole of an ideal gas at STP (1 atm and 273.15 K) is 22.414 \( \text{L} \). Substituting these values and solving the ideal gas law equation for \( R \) gives the following.

\[
R = \frac{PV}{nT} = \frac{(1 \text{ atm})(22.414 \text{ L})}{(1 \text{ mol})(273.15 \text{ K})} = 0.08205784 \text{ L\cdot atm/mol\cdot K}
\]

This calculated value of \( R \) is usually rounded to 0.0821 \( \text{L\cdot atm/(mol\cdot K)} \). Use this value in ideal gas law calculations when the volume is in liters, the pressure is in atmospheres, and the temperature is in kelvins. See Figure 3.4 for the value of \( R \) when other units for \( n \), \( P \), \( V \), and \( T \) are used.

Finding \( P \), \( V \), \( T \), or \( n \) from the Ideal Gas Law

The ideal gas law can be applied to determine the existing conditions of a gas sample when three of the four variables, \( P \), \( V \), \( T \), and \( n \), are known. It can also be used to calculate the molar mass or density of a gas sample.

Be sure to match the units of the known quantities and the units of \( R \). In this book, you will be using \( R = 0.0821 \text{ L\cdot atm/(mol\cdot K)} \). Your first step in solving any ideal gas law problem should be to check the known values to be sure you are working with the correct units. If necessary, you must convert volumes to liters, pressures to atmospheres, temperatures to kelvins, and masses to numbers of moles before using the ideal gas law.

### Figure 3.4

#### Numerical Values of Gas Constant, \( R \)

<table>
<thead>
<tr>
<th>Units of ( R )</th>
<th>Numerical value of ( R )</th>
<th>Unit of ( P )</th>
<th>Unit of ( V )</th>
<th>Unit of ( T )</th>
<th>Unit of ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\text{L\cdot mm Hg}}{\text{mol\cdot K}} )</td>
<td>62.4</td>
<td>mm Hg</td>
<td>L</td>
<td>K</td>
<td>mol</td>
</tr>
<tr>
<td>( \frac{\text{L\cdot atm}}{\text{mol\cdot K}} )</td>
<td>0.0821</td>
<td>atm</td>
<td>L</td>
<td>K</td>
<td>mol</td>
</tr>
<tr>
<td>( \frac{\text{J}}{\text{mol\cdot K}} )</td>
<td>8.314</td>
<td>Pa</td>
<td>m(^3)</td>
<td>K</td>
<td>mol</td>
</tr>
<tr>
<td>( \frac{\text{L\cdot kPa}}{\text{mol\cdot K}} )</td>
<td>8.314</td>
<td>kPa</td>
<td>L</td>
<td>K</td>
<td>mol</td>
</tr>
</tbody>
</table>

Note: 1 L\cdot atm = 101.325 J; 1 J = 1 Pa\cdot m\(^3\)

*SI units
Using the Ideal Gas Law

**Sample Problem I** What is the pressure in atmospheres exerted by a 0.500 mol sample of nitrogen gas in a 10.0 L container at 298 K?

1. **ANALYZE**
   - **Given:**
     - \( V \) of \( N_2 \) = 10.0 L
     - \( n \) of \( N_2 \) = 0.500 mol
     - \( T \) of \( N_2 \) = 298 K
   - **Unknown:** \( P \) of \( N_2 \) in atm

2. **PLAN**
   - \( n, V, T \rightarrow P \)
   - The gas sample undergoes no change in conditions. Therefore, the ideal gas law can be rearranged and used to find the pressure as follows:
     \[ P = \frac{nRT}{V} \]

3. **SOLVE**
   - \[ P = \frac{(0.500 \text{ mol}) \times (0.0821 \text{ L} \cdot \text{atm/} \text{mol} \cdot \text{K}) \times (298 \text{ K})}{10.0 \text{ L}} = 1.22 \text{ atm} \]

4. **CHECK YOUR WORK**
   - All units cancel correctly to give the result in atmospheres. The answer is properly limited to three significant figures. It is also close to an estimated value of 1.5, computed as \((0.5 \times 0.1 \times 300)/10\).

**Practice** Answers in Appendix E

1. What pressure, in atmospheres, is exerted by 0.325 mol of hydrogen gas in a 4.08 L container at 35°C?
2. A gas sample occupies 8.77 L at 20°C. What is the pressure, in atmospheres, given that there are 1.45 mol of gas in the sample?

---

**SECTION 3 FORMATIVE ASSESSMENT**

**Reviewing Main Ideas**

1. State Avogadro’s law, and explain its significance.
2. What volume (in milliliters) at STP will be occupied by 0.0035 mol of methane, \( CH_4 \)?
3. State the ideal gas law equation, and tell what each term means.
4. What would be the units for \( R \) if \( P \) is in pascals, \( T \) is in kelvins, \( V \) is in liters, and \( n \) is in moles?
5. A 4.44 L container holds 15.4 g of oxygen at 22.55°C. What is the pressure?
6. A tank of hydrogen gas has a volume of 22.9 L and holds 14.0 mol of the gas at 12°C. What is the pressure of the gas in atmospheres?

**Critical Thinking**

7. **ANALYZING DATA** Nitrous oxide is sometimes used as a source of oxygen gas:
   \[
   2N_2O(g) \rightarrow 2N_2(g) + O_2(g)
   \]
   What volume of each product will be formed from 2.22 L \( N_2O \)? At STP, what is the density of the product gases when they are mixed?